# Construction of one-dimensional photonic crystals based on the incident angle domain

Biqin Huang, Peifu Gu, and Ligong Yang

State Key Laboratory for Modern Optical Instrumentation, Zhejiang University, Yugu Road, Hangzhou 310027,

People's Republic of China

(Received 8 May 2003; published 7 October 2003)

An effective method used to extend the band gap of hybrid one-dimensional photonic crystals is proposed. This method does not require one-dimensional photonic crystals used in the construction of hybrid photonic crystals to have omnidirectional photonic band gaps. This property gives the designers more freedom to select materials used in the design. Another advantage of this method is the ability to obtain very large omnidirectional photonic band gaps of hybrid structures without using too many one-dimensional photonic crystals. Some structures are constructed by this method to prove it. By using  $TiO_2$  and  $SiO_2$ , the structure composed of four one-dimensional photonic crystals based on this method has been calculated and fabricated. The theoretical relative bandwidth of an omnidirectional photonic band gap is 41.33%, and the experimental result is up to 37%, which is very large in the visible region. By comparison, the theoretical relative bandwidth of the conventional method based on the frequency domain is only 14.26%.

DOI: 10.1103/PhysRevE.68.046601

PACS number(s): 42.70.Qs, 51.70.+f

### I. INTRODUCTION

For many years scientists have expected to control light propagation as freely as possible. But it seemed impossible to do it until 1987, when Yablonovitch and John [1,2] introduced the photonic crystals to the world. After that, photonic crystals became the focus of many theorists' and experimenters' work. In order to obtain the complete photonic band gap, a three-dimensional photonic crystal is necessary. Nevertheless, the existent technology cannot fabricate such small three-dimensional structures, especially for the photonic crystals in the visible region. So researchers have turned to fabricate low-dimensional photonic crystals. About 1998, the work of one-dimensional photonic crystals has attracted much attention [3-6]. They indicated that there exists a range of frequencies in one-dimensional photonic crystals where light from arbitrary angles cannot propagate in the crystals. This property is useful to fabricate dielectric mirrors, which are better than metal mirrors owing to their extremely low loss. But more importantly, recently this property is used to fabricate the omniguide fiber, which can guide optical light in air [7-10]. This type of fiber is similar to the photonic crystal fiber with air holes. Its appearance will bring great changes to light transmission. In these applications the width of omnidirectional photonic band gap is the most important.

The width of an omnidirectional band gap mostly depends on the refraction index of the materials used to construct the photonic crystals. Generally, the higher the ratio of the high to the low refraction index, the larger the relative bandwidth of band gap. However, the refraction index of the lossless materials which are suitable for optical applications are usually low, especially in the visible region. And the ratio of refractive index is not very high. So the relative bandwidth that can be acquired in the 1D PC (one-dimensional photonic crystal) is not very large. For the applications in visible light, the band gap that can be obtained is usually not larger than 10%. This is not enough for many applications. In order to get larger relative bandwidth, many methods have been proposed [11-14]. In literature [12], the disordered 1D photonic crystals are used to extend the band gap. But parameters of the structure are irregular. So the fabrication becomes relatively difficult. A promising method is suggested in literature [14]. By using some 1D photonic crystals whose omnidirectional band gap overlap each other, the relative bandwidth is extended. From the view of theory, an arbitrary bandwidth can be obtained using enough 1D photonic crystals. But as indicated above, the band gap of each 1D photonic crystal is usually narrow, so a broad band gap cannot be acquired unless there are many overlapping 1D photonic crystals. What is most important is that this method requires each 1D photonic crystal to have an omnidirectional photonic band gap. This requirement prevents many materials from being used in the fabrication of photonic crystals. So materials that can be used in the fabrication are scarce, especially for the fabrication of omniguide fiber, which is stricter for the selection of materials [9,10].

In this paper, we first review the method based on the frequency domain used in literature [14] in Sec. II. Then in Sec. III, a more effective and practical method based on incident angle domain is proposed. This method arises from a very different view in contrast with the method in literature [14]. In this method the requirement for the refraction index of materials is very low. It does not require 1D photonic crystals with omnidirectional band gaps to be used in the extension of an omnidirectional photonic band gap. This allows many materials with good properties suitable to be used in many applications. The experimental verification of this method is demonstrated in Sec. IV. The result of the experiment indicates that the method can extend the omnidirectional band gap remarkably.

#### **II. METHOD BASED ON FREQUENCY DOMAIN**

The method used in literature [14] is shown in the Fig. 1. PC<sub>1</sub> (photonic crystals 1) has omnidirectional band gap from angular frequency  $\omega_1$  to  $\omega_2$ . PC<sub>2</sub>'s omnidirectional band gap is from  $\omega_2$  to  $\omega_3$ . So the whole component's omnidirectional band gap should be from  $\omega_1$  to  $\omega_3$ . Apparently, the



FIG. 1. The sketch map for the method based on frequency domain.

omnidirectional band gap can be extended. And the relative bandwidth of this structure can be expressed as  $2(\omega_3 - \omega_1)/\omega_3 + \omega_1$ . Because each 1D PC in this method should have omnidirectional photonic band gap, this method can be thought of as the construction of photonic crystals based on frequency domain. It is easy to prove that

$$\frac{2(\omega_3-\omega_1)}{\omega_3+\omega_1} < \frac{2(\omega_2-\omega_1)}{\omega_2+\omega_1} + \frac{2(\omega_3-\omega_2)}{\omega_3+\omega_2}$$

This expression indicates that the relative bandwidth of composed structures is smaller than the sum of the relative bandwidths of each 1D PC used in this method. Take TiO<sub>2</sub> and SiO<sub>2</sub>, for example, and assume that their refraction indices are 2.4 and 1.46, respectively. The filling ratio of TiO<sub>2</sub> is 1.46/(1.46+2.4), which means that the optical thickness of layers is equal to each other. As stated in Ref. [15], the band gap of one-dimensional photonic crystal can be calculated by the equation

$$\cos K\Lambda = \cos(k_{1x}a)\cos(k_{2x}b) - M\sin(k_{1x}a)\sin(k_{2x}b),$$
(1)

$$M = \begin{cases} \frac{1}{2} \left( \frac{k_{2x}}{k_{1x}} + \frac{k_{1x}}{k_{2x}} \right), & \text{TE} \\ \frac{1}{2} \left( \frac{n_1^2 k_{2x}}{n_2^2 k_{1x}} + \frac{n_2^2 k_{1x}}{n_1^2 k_{2x}} \right), & \text{TM}, \end{cases}$$
(2)

where *K* is the Bloch wave vector.  $k_{1x}$  and  $k_{2x}$  are the normal components of the wave vectors in the two dielectric layers,  $n_1$  and  $n_2$  are their refraction indices, respectively, *a* and *b* are their physical thicknesses and  $\Lambda$  is the structure period. So the omnidirectional photonic band gap for such a structure is from 0.3073 to 0.3185, which is normalized by  $2\pi C/\Lambda$ . *C* is the speed of light in vacuum. All of the angular frequencies in this paper have been normalized in this way. And relative bandwidth is 3.6%. Based on this method, the structures composed of four 1D PCs whose parameters are listed above can be obtained by adjusting the periods of each 1D PC appropriately. When the ratio of each 1D PC's period is equal to 1:1.0364:1.0741:1.1132, the omnidirectional pho-



FIG. 2. The dispersion relation between angular frequency and wave vector parallel to the surface.

tonic band gap of this hybrid structure is from 0.2761 to 0.3185. And the relative bandwidth is 14.26%, which is smaller than 14.4. From these discussions, we can see that the relative bandwidth of constructed structures is determined by the relative bandwidth of each 1D PC and the number of 1D PCs used in it. In order to obtain large band gap, it is very important to select appropriate materials. Normally, the ratio of refraction index between high and low refraction index materials should be very high. So whether the construction of photonic crystals based on this method is successful or not is determined greatly by the materials used in it.

### **III. METHOD BASED ON INCIDENT ANGLE DOMAIN**

Normally, the omnidirectional band gap of 1D photonic crystals requires two criteria-the frequency and the incident angles. The 1D PC used in the method based on frequency domain satisfies both the criteria. The dispersion relation of angular frequency and wave vector parallel to the surface for  $TiO_2$  and  $SiO_2$  with equal optical thickness, is shown in Fig. 2. The left part of the figure represents the TM mode and the right part stands for the TE mode. It is easy to find that the omnidirectional photonic band gap is from  $\omega_2$  to  $\omega_3$ . At the same time, we can find that if the incident angle is limited from  $0^{\circ}$  to  $60^{\circ}$ , there also exists one corresponding band gap from  $\omega_1$  to  $\omega_3$ . The band gap from  $\omega_1$  to  $\omega_3$  can be termed partial band gap for  $0^{\circ}$  to  $60^{\circ}$ , in contrast with the omnidirectional band gap requiring the incident angle from  $0^{\circ}$  to 90°. It is the same for the incident angle from  $60^{\circ}$  to  $90^{\circ}$ , whose partial band gap is from  $\omega_2$  to  $\omega_4$ . If we consider only the band gap of one 1D PC for the incident angles from zero to some degree  $\theta$ , for example, 50°, and the other one from  $50^{\circ}$  to  $90^{\circ}$ , through adjusting their periods these two 1D PCs' partial band gap can overlap each other to obtain the omnidirectional photonic band gap. As shown in Fig. 3, the PC<sub>1</sub> has band gap  $(\omega_3 - \omega_1)$  for the partial angle range (0  $-\theta$ ). But the frequency from  $\omega_3$  to  $\omega_2$  is within the omni-



FIG. 3. The sketch map of the method based on the incident angle domain.

directional band gap. So the real partial band gap is from  $\omega_2$  to  $\omega_1$ . It is the same for the PC<sub>2</sub>. Now by adjusting the period of each PC, the partial band gap of each PC can overlap each other. For example, if  $\omega_5$  is equal to  $\omega_1$ , and  $\omega_6$  is not larger than  $\omega_2$ , the omnidirectional band gap of the constructed structure is from  $\omega_3$  to  $\omega_4$ . It is evident that its relative bandwidth will be larger than the hybrid structure designed by the method in Ref. [14], because the omnidirectional band gap of each 1D PC.

In what follows, the performance of structures composed of four 1D PCs based on this method will be investigated. The materials used here are also  $TiO_2$  and  $SiO_2$  and parameters of each 1D PC are the same as those mentioned above. In Fig. 4, the relative bandwidth at different incident angle ranges is presented. Line 1 represents relative bandwidth as the function of the lower limit of incident angles, while the higher limit of incident angles is constant, equal to 90°. On the other hand, line 2 represents the relative bandwidth as the function of the higher limit of incident angles, while the lower limit of incident angles is always zero. The parameters of 1D PC are the same as that in Fig. 2. So in order to make the partial band gap of each PC overlap each other well, the incident angles for partial band gap should be chosen appropriately. Here it is chosen from  $0^{\circ}$  to  $56.7^{\circ}$  to  $90^{\circ}$ . The ratio of periods between four 1D PCs is equal to 1:1.1363:1.2914:1.4674. The calculated omnidirectional photonic band gap is from 0.2094 to 0.3185 and the relative bandwidth is about 41.33%, which is more than two times of the relative bandwidth of the structure based on the method in literature [14]. The calculated reflection of this structure at different incident angles through transfer-matrix method is shown in Fig. 5. Each 1D PC in calculation is composed of 12 cells, i.e., 24 layers. The periods of each 1D PC are 137.7, 156.5, 177.8, and 202.1 nm, respectively. Considering the frequency where reflection is higher than 99% to be forbidden, the omnidirectional band gap is from 437 to 652 nm and its relative bandwidth is about 39.49%, which is close to the theoretical design for the infinite period structure. If we calculate the infinite structures, the result of the calculation will be closer to it.

From the discussion above, it is evident that based on this method, the relative bandwidth of hybrid 1D photonic crystals can be extended remarkably. But what is the most promising property of this method is that it does not require that 1D PCs used in it have omnidirectional photonic band gap. This means that the ratio of high- and low-index materials is not so severe. Take ZnO and SiO<sub>2</sub> for example. Their refraction indices are 2.1 and 1.46, respectively. Their optical thickness is equal. The dispersion relation is shown in Fig. 6. It shows that there does not exist omnidirectional photonic band gap. But partial band gap does exist, which is from  $\omega_1$  to  $\omega_2$  for incident angle 0°-60°. If incident angles are chosen from 0°-56° and from 56°-90°, the corresponding partial band gaps are from 0.3065 to 0.3237 and from 0.3411 to



FIG. 4. The relative bandwidth of partial band gap for different incident angle ranges for the 1D PC composed TiO<sub>2</sub> and SiO<sub>2</sub>.



FIG. 5. The calculated reflection of hybrid 1D PC composed of four 1D PCs at different incident angles 0°, of 60°, and 80°.

0.3600. Their relative bandwidths are 5.44% and 5.39%. By adjusting the periods of two 1D PCs, the omnidirectional band gap of hybrid 1D PC will be 5.39%. Its calculated reflection is shown in Fig. 7. Each 1D PC has 38 layers. The purpose to take so many layers is to get more accurate result. Another reason is that the relative band gap of this hybrid structure is small. The periods of 1D PC are 174.2 and 193.7 nm, respectively. It can be found that the omnidirectional band gap of this structure is from 539 to 563 nm, whose relative bandwidth is about 4.36%.

From these discussions, it can be seen that based on this method, the designer will have more freedom to select the appropriate materials to fabricate a better performance device based on omnidirectional photonic band gap effect.



FIG. 6. The dispersion between angular frequency and wave vector parallel to the surface for the 1D PC composed of ZnO and  $SiO_2$  with equal optical thickness.



FIG. 7. Calculated reflection of hybrid 1D PC composed two 1D PCs at different incident angles 0°, of 60°, and 80°.

#### **IV. EXPERIMENTAL VERIFICATION**

Thin film technology is used in this experiment due to its convenience. In order to get high reflection of hybrid 1D PC, the number of layers in each 1D PC should be adequate. So the number of layers for the whole hybrid 1D PC will be large and the thickness of the thin film is normally determined by the properties of materials. To deposit so many layers on substrates, it is necessary to choose some appropriate materials. TiO<sub>2</sub> and SiO<sub>2</sub> are good candidates, because they have been extensively studied in thin film technology. Their properties are familiar to scientists. And the most important is that these two materials are very suitable to make very thick films. The materials are deposited by using an electron-beam evaporator at vacuum condition of 2.5  $\times 10^{-3}$  Pa and substrate temperature 300°C. In order to get high refraction index for TiO<sub>2</sub>, ion assisted deposition is employed in this experiment. Its energy is about 200 eV and anode current is about 3 A. The refraction indices of  $TiO_2$ and SiO<sub>2</sub> are expected to be 2.4 and 1.46. Control wavelengths of each 1D PC are 500, 568, 645, and 733 nm, respectively. The whole hybrid 1D PC has  $4 \times 24$  layers. The scheme of this structure is shown in Fig. 8. The calculated reflection is shown in Fig. 5. Figure 9 shows its measured transmission at different incident angles 0°, 60°, and 80°. It is measured in the spectrometer UV-3101 PC by adding a rotary table. The omnidirectional band gap is from 502 to



FIG. 8. The scheme of hybrid photonic crystal used in the experimental fabrication. H and L are TiO<sub>2</sub> and SiO<sub>2</sub>, respectively. The designed thicknesses of TiO<sub>2</sub> and SiO<sub>2</sub> for PC<sub>1</sub> are 52.08 and 85.62 nm, respectively. For PC<sub>2</sub> they are 59.17 and 97.26 nm. For PC<sub>3</sub> they are 67.19 and 110.45 nm. For PC<sub>4</sub> they are 76.35 and 125.51 nm.



FIG. 9. The measured transmission of hybrid 1D PC composed of four 1D PCs designed by the method based on incident angle domain. In each 1D PC, the optical thickness of each layer is equal. The incident angles are  $0^{\circ}$ ,  $60^{\circ}$ , and  $80^{\circ}$ , respectively.

735 nm. The relative bandwidth is about 37%. It can be seen that there exist some frequencies where transmission is a bit high. The reason is that in this experiment, we are not able to fabricate the infinite number of layers. If the number of layers is increased, the final result will be expected to be better than this. Another reason is that the refraction indices of materials are dependent on the frequency and it is not considered in the calculation of the band structures. To make the experimental result agree with the theoretical calculation, the dispersion effect should be included. Nevertheless, this result is able to prove that the method can extend the omnidirectional band gap greatly. From Fig. 9 we can find that the

- [1] E. Yablonovitch, Phys. Rev. Lett. 58, 2059 (1987).
- [2] S. John, Phys. Rev. Lett. 58, 2486 (1987).
- [3] Yoel Fink, Joshua N. Winn, Shanhui Fan, Chiping Chen, Jurgen Michel, John D. Joannopoulos, and Edwin L. Thomas, Science 282, 1679 (1998).
- [4] Joshua N. Winn, Yoel Fink, Shanhui Fan, and J. D. Joannopoulos, Opt. Lett. 23, 1573 (1998).
- [5] D. N. Chigrin, A. V. Lavrinenko, D. A. Yarotsky, and S. V. Gaponenko, Appl. Phys. A: Mater. Sci. Process. A68, 25 (1999).
- [6] J. P. Downling, Science 282, 1841 (1998).
- [7] Yoel Fink, Daniel J. Ripin, Shanhui Fan, Chiping Chen, John D. Joannopoulos, Edwin L. Thomas, J. Lightwave Technol. LT17, 2039 (1999).
- [8] M. Ibanesce, Y. Fink, S. Fan, E. L. Thomas, and J. D. Joannopoulos, Science 289, 415 (2000).

partial band gap for  $0^{\circ}-60^{\circ}$  is from 502 to 799 nm, whose relative bandwidth is 45.66%. It is easy to calculate that the relative bandwidth of hybrid 1D PC composed of four 1D PCs (based on the method in literature [14]) is about 22.81%. It shows that the partial band gap also has been extended. Of course, if the hybrid structures are designed to extend the partial band gap for  $0^{\circ}-60^{\circ}$ , the final result will be greater than 45.46%.

## V. CONCLUSION

From the aforementioned discussion, a practical and effective method used to design hybrid one-dimensional photonic crystals has been proposed. By this method the relative bandwidth can be extended remarkably, while the structure is not very complex and easy to be fabricated. Based on this method a hybrid structure has been fabricated. It shows large omnidirectional band gap and fully supports the theory. This method can be used to design the structures for applications needing a broad omnidirectional band gap. In fact, this method is suitable to optimize the partial band gap of structures for a fixed incident angle range. What is the most promising is that this method does not require one-dimensional photonic crystals to be used to construct hybrid structures with an omnidirectional band gap. What is needed are photonic crystals with partial band gaps. This relaxed requirement makes this method very useful to design some structures based on some special materials. A pure onedimensional photonic crystal based on these materials is not able to present an omnidirectional band gap. This method strongly broadens the selection of materials.

This work was supported by the National Nature Science Foundation of China Grant No. (6007800).

- [9] Shandon D. Hart, Garry R. Maskaly, Burak Temelkuran, Peter H. Prideaux, John D. Joannopoulos, and Yoel Fink, Science 296, 510 (2002).
- [10] Burak Temelkuran, Shandon d. Hart, Gilles Benolt, John D. Joannopoulos, and Yoel Fink, Nature (London) 420, 650 (2002).
- [11] E. Yablonovitch, Opt. Lett. 23, 1648 (1998).
- [12] Hongqiang Li, Hong Chen, Xinjie Qiu, Physica B 279, 164 (2000).
- [13] A. Mir, A. Akjouj, E. H. El Boudouti, B. Djafari-Rouhani, and L. Dobrzynski, Vacuum 63, 197 (2001).
- [14] Xin Wang, Xinghua Hu, Yizhou Li, Wulin Jia, Chun Xu, Xiaohan Liu, and Jian Zi, Appl. Phys. Lett. 80, 4291 (2002).
- [15] Amnon Yariv and P. Yeh, Optical Waves in Crystals (Wiley, New York, 1984).